

Twisted reduction in large N QCD with adjoint Wilson fermions

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The twisted reduced model of large N QCD with two adjoint Wilson fermions is studied numerically using the Hybrid Monte Carlo method. This is the one-site model, whose large N limit (large volume limit) is expected to be conformal or nearly conformal. The string tension calculated at $N=289$ approaches zero as we decrease quark mass and the preliminary value of the mass anomalous dimension γ_* is close to one if we assume that the theory is governed by an infrared fixed point. We also discuss the twisted reduced model with single adjoint Wilson fermion. The string tension remains finite as the quark mass decreases to zero, supporting that this is the confining theory.

Keywords: large N QCD, twisted space-time reduction, adjoint fermion

1. Introduction

The standard model of the elementary particle has set the foundation to $SU(N)$ gauge theories. Although $SU(N)$ gauge theories generally have very complicated structures, significant simplifications could occur in the large N limit. In fact, Eguchi and Kawai considered the space-time reduced model of the lattice gauge theory having only one space-time point.¹ This model is now called the Eguchi-Kawai model (EK-model). The EK model possesses the $Z(N)$ symmetry. If this symmetry is not spontaneously broken, the lattice gauge theory and the EK model are equivalent in the large N limit. This means that space-time degrees of freedom of the lattice gauge theory could be encoded into those of internal $SU(N)$ group. Soon after the proposal of Eguchi and Kawai, however, the $Z(N)$ symmetry was shown to be spontaneously broken in the weak coupling limit, thus invalidating the EK model.² To circumvent this difficulty, the present authors have proposed to consider the one-site model with twisted boundary conditions (the twisted Eguchi-Kawai (TEK) model), thus avoiding the $Z(N)$ symmetry being spontaneously broken.^{3,4} Recently, gauge theories coupled with fermions in the adjoint representation received much attention

in the large N limit in connection, for example, to the AdS/CFT correspondence. The idea of the twisted space-time reduction can also be applied to these models. The purpose of this talk is to present our recent progress in this direction. In the next section, we calculate the continuum string tension of the $SU(N)$ pure gauge theory in the large N limit using the TEK model. In sect. 3, large N QCD with two adjoint Wilson fermions is studied with the twisted space-time reduction, and sect. 4 is devoted to the large N QCD with single adjoint Wilson fermion. Conclusions are given in sect. 5.

2. The TEK model

We consider the $SU(N)$ group with $N = L^2$, L being some positive integer. Then the action of the TEK model is obtained from the action of the $SU(N)$ lattice gauge theory

$$S = -bN \sum_n \sum_{\mu \neq \nu=1}^4 \text{Tr} \left[U_{n,\mu} U_{n+\mu,\nu} U_{n+\nu,\mu}^\dagger U_{n,\nu}^\dagger \right] \quad (1)$$

by neglecting the space-time dependence of the link variable $U_{n,\mu} \rightarrow U_\mu$ and multiplying the twist tensor $Z_{\mu\nu}$ in front of the plaquette action as³

$$S_{TEK} = -bN \sum_{\mu \neq \nu=1}^4 \text{Tr} \left[Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right] \quad (2)$$

As a result, there remain only four $SU(N)$ matrices U_μ . The inverse 't Hooft coupling is labelled $b = 1/g^2 N$. The elements of the twist tensor belong to $Z(L)$ and the explicit form is given by

$$Z_{\mu\nu} = \exp \left(k \frac{2\pi i}{L} \right), \quad Z_{\nu\mu} = Z_{\mu\nu}^*, \quad \mu > \nu \quad (3)$$

The action S_{TEK} is invariant under the $Z(L)$ transformation $U_\mu \rightarrow z U_\mu$, $z \in Z(L)$. If this symmetry is not spontaneously broken, the TEK model and the corresponding lattice gauge theory are equivalent in the large N limit. k and L should be coprime, and a general prescription for choosing k and L to preserve the $Z(L)$ symmetry has been given in ref. [4]. In this talk, we choose $L=29$ and $k=11$ for the TEK model.^{5,6} For finite L , the TEK model is closely related to the lattice gauge theory on a finite space-time volume L^4 , and we are able to calculate Wilson loops up to size 14×14 for $L=29$. In the TEK model, the Wilson loop $W(R, T)$ with size $R \times T$ is obtained from four link variables U_μ as

$$W(R, T) = Z_{\mu\nu}^{RT} < U_\mu^R U_\nu^T U_\mu^{R\dagger} U_\nu^{T\dagger} >. \quad (4)$$

Then the string tension σ is extracted from the large distance behavior of the Creutz ratio

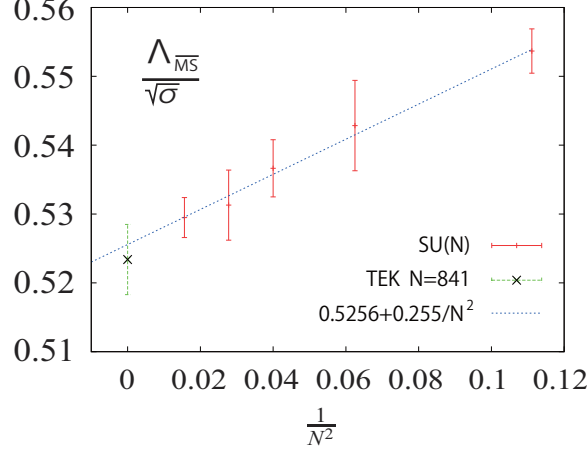


Fig. 1. N dependence of the continuum string tension.

$$\chi(R', T') = -\log \frac{W(R' + 0.5, T' + 0.5)W(R' - 0.5, T' - 0.5)}{W(R' + 0.5, T' - 0.5)W(R' - 0.5, T' + 0.5)} \quad (5)$$

as

$$\chi(R', R') = \sigma + \frac{2\gamma}{R'^2} + \frac{\eta}{R'^4} \quad (6)$$

with half-integer R' and T' . We calculate the continuum string tension by extrapolating the TEK data at six values of b . For comparison, we also calculate the continuum string tension of the usual $SU(N)$ lattice gauge theory with $N = 3, 4, 5, 6, 8$ on a $V = 32^4$ lattice. Results are summarized in Fig. 1. The \times point is $\Lambda_{\overline{MS}}/\sqrt{\sigma}$ for the TEK model with $N=841$. The $+$ symbols are results for the lattice gauge theory, and dotted line is a linear fit of these data as a function of $1/N^2$. The large N extrapolated value agrees remarkably well with the result of the TEK model, demonstrating the correctness of the twisted space-time reduction idea.

3. Large N QCD with two adjoint fermions

$SU(N)$ gauge theories with two adjoint fermions are thought to be conformal or nearly conformal for any value of N , since the first two coefficients of the beta function expressed in term of 't Hooft coupling are independent of N , namely, $b_0 = (4N_f - 11)/24\pi^2$ and $b_1 = (16N_f - 17)/192\pi^4$. For $N_f = 2$, $b_0 < 0$ and $b_1 > 0$, then we naturally expect that there is a infrared fixed point for finite value of 't Hooft coupling. In fact, for $N=2$ (minimal walking technicolor), there are now many lattice simulations indicating that the theory is conformal at vanishing fermion mass.⁷ The mass anomalous dimension, however, is shown to be rather small $\gamma_* \sim 0.3$, to explain the large values of the observed quark masses, within the walking technicolor scenario.

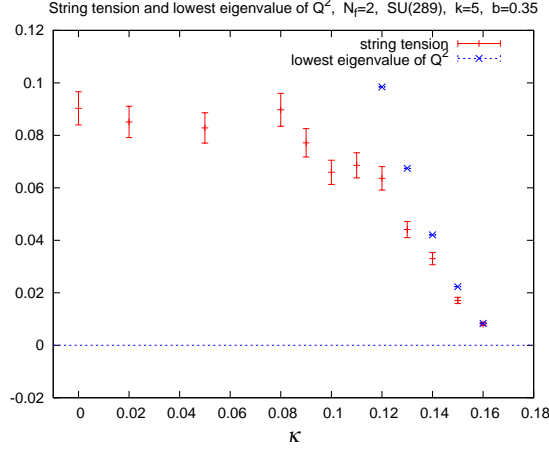


Fig. 2. String tension and lowest eigenvalue of Q^2 at $b=0.35$ with $N_f = 2$.

We can also consider the twisted space-time reduced model of large N QCD with two adjoint fermions.^a The action reads

$$S = -bN \sum_{\mu \neq \nu=1}^4 \text{Tr} [Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger] + \sum_{j=1}^{N_f} \bar{\Psi}_j D_W \Psi_j \quad (7)$$

with $N_f=2$ and the Wilson-Dirac operator D_W given by

$$D_W = 1 - \kappa \sum_{\mu=1}^4 [(1 - \gamma_\mu) U_\mu^{adj} + (1 + \gamma_\mu) U_\mu^{\dagger adj}] \quad (8)$$

Ψ_j is the fermion matrix in the color (N, \bar{N}) representation. Thus the link variable in the adjoint representation U_μ^{adj} actually acts on Ψ_j as $U_\mu^{adj} \Psi_j = U_\mu \Psi_j U_\mu^\dagger$. The hopping parameter of the fermion field κ is related to the bare fermion mass as $m_f = (1/2)(1/\kappa - 1/\kappa_c)$ with κ_c the critical hopping parameter.

We calculate the string tension at two values of b , 0.35 and 0.36, and for various values of κ . We are using $N = 289 = 17^2$ and $k = 5$, which should compare with ordinary lattice theory with $V = 17^4$.¹¹ For $N_f = 2$, we can use the standard hybrid Monte Carlo method. Simulations have been done on Hitachi SR16000 supercomputer at KEK having peak speed of 980 GFlops per node. Thanks to the Hitachi system engineers, our code is highly optimized for the SR16000. The sustained speed of our code being 600 GFlops.

^aFor previous works of the space-time reduced model of large N QCD with adjoint fermions, see ref. [8–10].

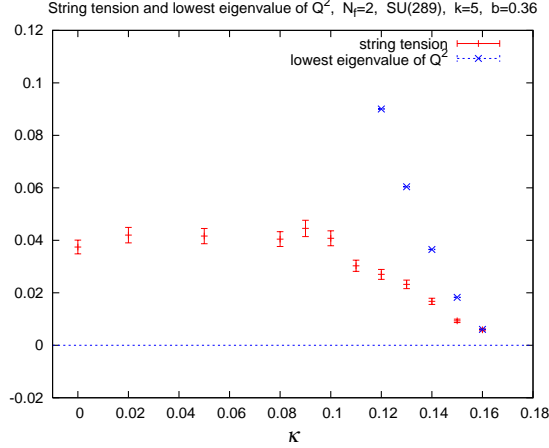


Fig. 3. String tension and lowest eigenvalue of Q^2 at $b=0.36$ with $N_f = 2$.

In Fig. 2, we show the string tension σ as a function of κ at $b=0.35$ with $+$ symbols. The value of σ at $\kappa=0$ is obtained from the TEK model without fermions. We clearly see that as κ increases the string tension rapidly decreases and seems to vanish at $\kappa \sim 0.17$. So far, we have not calculated any hadronic spectrum. It is quite straightforward, however, to calculate the lowest eigenvalue of the positive hermitian Wilson-Dirac operator $Q^2 = (D_W \gamma_5)^2$, which should be related to the physical fermion mass square. In Fig. 2, we also show the lowest eigenvalue of Q^2 as \times symbols. The string tension and the lowest eigenvalue of Q^2 vanish simultaneously, which strongly supports that the string tension is zero at critical κ_c where the fermion mass vanishes.

We can fit both the string tension σ and the lowest eigenvalue of Q^2 with the same fitting form $a(1/\kappa - 1/\kappa_c)^b$. Requiring that both quantities vanish at the same critical κ_c , we have $\kappa_c = 0.1694(7)$ and, for the string tension, $b = 1.10(4)$. If the massless theory is governed by an infrared fixed point, b is related to the mass anomalous dimension γ_* as $b = 2/(1 + \gamma_*)$, then we have $\gamma_* = 0.81(8)$.

We have repeated the analysis at $b=0.36$, a little bit closer to the continuum limit. Results are shown in Fig. 3. It seems that the data at $\kappa = 0.16$ are largely affected by the finite -size effects. Excluding the data at $\kappa = 0.16$ from our analysis, we have $\kappa_c = 0.168(1)$, $b = 1.33(4)$, and thus $\gamma_* = 1.17(21)$. Although our data are still preliminary, large N QCD with two adjoint fermions seems to be a conformal field theory with large mass anomalous dimension $\gamma_* \sim 1$.

4. Large N QCD with single adjoint fermion

In the large N limit, $N_f = 1$ adjoint fermion is equivalent to the $N_f = 2$ fundamental fermions in the rank two anti-symmetric representation (the orientifold equivalence⁸). For $N = 3$, the latter theory is just the two flavor QCD. This means

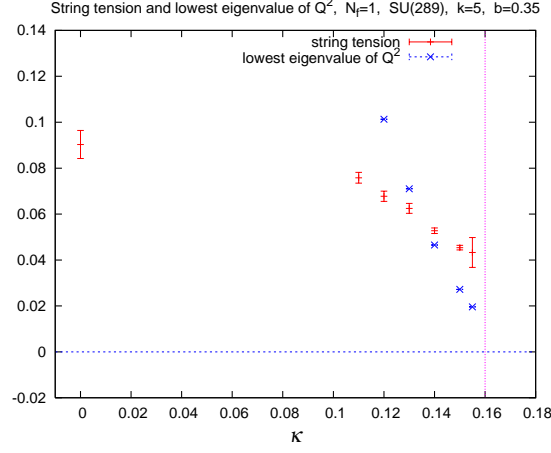


Fig. 4. String tension and lowest eigenvalue of Q^2 at $b=0.35$ with $N_f = 1$.

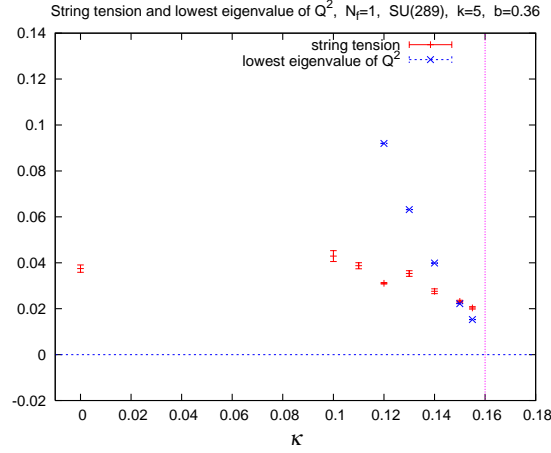


Fig. 5. String tension and lowest eigenvalue of Q^2 at $b=0.36$ with $N_f = 1$.

that the large N QCD with single adjoint fermion corresponds to the Corrigan-Ramond large N limit¹² of realistic QCD, and thus it is quite interesting to analyze this theory from the phenomenological point of view.

We use the rational hybrid Monte Carlo method to simulate the $N_f = 1$ theory. In Fig. 4, we plot the string tension σ as a function of κ at $b=0.35$ with + symbols. Also shown is the lowest eigenvalue of Q^2 with \times symbols. The lowest eigenvalue of Q^2 seems to vanish around $\kappa \sim 0.165$, while the string tension seems to remain finite at this point, indicating that the large N QCD with single adjoint fermion is a confining theory. This is quite natural since our theory should be related to the

two flavor $N = 3$ QCD. We found that, for $\kappa \geq 0.16$, we cannot make simulations since the CG iteration does not converge, which is also the common phenomena encountered in the simulation of QCD.

In Fig. 5, we show the results at $b=0.36$. The absolute values of the string tension are smaller than those of $b = 0.35$ since we are approaching to the continuum limit. The string tension, however, remains finite around $\kappa \sim 0.165$ as in Fig. 4. We also confirm that the CG iteration does not converge for $\kappa \geq 0.16$. Hence, both Figs. 4 and 5 indicate that the continuum theory of the large N QCD with single adjoint fermion is a confining theory.

5. Conclusion

We have demonstrated that the twisted reduction works quite well for the description of large N QCD. For pure gauge theory, we have succeeded in calculating the continuum string tension using TEK model. Although our calculations with dynamical fermions are still preliminary, we have also shown that large N QCD with two adjoint fermions is a conformal theory with large mass anomalous dimension $\gamma_* \sim 1$, and that the theory with single adjoint fermion is a confining theory. Since the rational hybrid Monte Carlo method we used for the simulation with one flavor can directly applicable to any number of flavor, we are planning to extend our analysis to $N_f = 1/2$ super-symmetric theory and also $N_f = 3/2$ and $5/2$ theories if they are conformal or confining theories.

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References

1. T. Eguchi and H. Kawai, *Phys. Rev. Lett.* **48** (1982) 1004.
2. G. Bhanot, U. M. Heller and H. Neuberger, *Phys. Lett.* **B113** (1982) 47.
3. A. Gonzalez-Arroyo and M. Okawa, *Phys. Rev.* **D27** (1983) 2397.
4. A. Gonzalez-Arroyo and M. Okawa, *JHEP* **1007** (2010) 043.
5. A. Gonzalez-Arroyo and M. Okawa, *PoS Lattice 2012* (2012) 221.
6. A. Gonzalez-Arroyo and M. Okawa, *Phys. Lett.* **B718** (2013) 1524.
7. L. Del Debbio, *PoS Lattice 2010* (2010) 004.
8. P. Kovtun, M. Unsal, L. G. Yaffe, *JHEP* **0706** 019 (2007) 019.
9. T. Azeyanagi, M. Hanada, M. Unsal, R. Yacoby, *Phys. Rev.* **D82** (2010) 125013.
10. B. Bringoltz, M. Koren, S. R. Sharpe, *Phys. Rev.* **D85** (2012) 094504.
11. A. Gonzalez-Arroyo and M. Okawa, *PoS Lattice 2012* (2012) 046.
12. E. Corrigan and P. Ramond, *Phys. Lett.* **B87** (1979) 73.